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Euler's method l'hopital's rule

↓

solve D.E. numerically

$$y_{n+1} = y_n + f'(x,y)\cdot \Delta x$$

new old

x	y	y'
0	0	1
.1	.1	1.2
.2	.22	1.44
.3	.364	1.6
.4	.524	1.8
.5	.704	

$y' = 2x+1$

$y(0) = 0$

$0 + 1(.1)$

$y(.5) = ?$

$.1 + 1.2(.1)$

use $\Delta x = .1$

$.22 + 1.44(.1)$

$.364 + 1.6(.1)$

$.524 + 1.8(.1)$

exact: $y(x) = x^2 + x + c$ init cond $c = 0$

$y(.5) = .5^2 + .5 = .75$

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l'hopital's rule

2 indet. forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

or $x \to \infty$

ex. 1 $\displaystyle \lim_{x\to\infty} \frac{\ln(x+1)}{\ln(3+x^2)} = \lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{3+x^2}\cdot 2x}$

$$\lim_{x\to\infty} \frac{1}{(x+1)} \frac{3+x^2}{2x} = \lim_{x\to\infty} \frac{3+x^2}{2x^2+2x} = \frac{1}{2}$$

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$$\frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$\lim_{x\to 0^+} \csc x - \cot x$$

$$\infty - \infty$$

$$\lim_{x\to 0^+} \frac{1-\cos x}{\sin x} \quad \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{\sin x}{\cos x} = 0$$

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$$\lim_{x\to\infty} \left(1+\frac{2}{3x}\right)^x = y \quad 1^\infty$$

$$\lim_{x\to\infty} \ln\left(1+\frac{2}{3x}\right)^x = \ln y$$

$$\lim_{x\to\infty} \overset{\infty}{x} \ln\overset{0}{\left(1+\frac{2}{3x}\right)} = \ln y$$

$$\frac{0}{0} \quad \lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{3x}\right)}{\frac{1}{x}} = \ln y$$

$$\frac{2}{3x} = \frac{2}{3}\cdot\frac{1}{x}$$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{2}{3x}} \cdot \frac{2}{3}\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}} = \frac{2}{3} = \ln y$$

$$y = e^{2/3}$$

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